$$GH(s) = \cancel{\frac{K(s+1)}{s(s+3)(s+4)}} \quad \frac{K(s+a)}{s(s+b)(s+c)}$$

Root locus

(a) Poles $\longrightarrow$ 3 Poles $\Rightarrow 0, -b, -c$

zeros $\longrightarrow$ 1 zero $\Rightarrow -a$

(b) Real Part

From

$0 \longrightarrow a$

$b \longrightarrow c$

مثلاً : تقف عند a وتبص عاليمين وتشوف ال Poles

، zero لو فردى يبق الجزء اللى قبلها (real part)

لـ زى من o ← a

4 Breaking Point

$1 + GH(s) = 0$ $\qquad S_b = \sim$

$GH(s) = 0$ $\qquad K \text{ at } S_b = \sim$

$K = \sim$

خطوة ال (breaking point) إستخدمناها

$\frac{dK}{ds} = 0$ لحدوث تصادم بسبب الجذرين b ، c .

$\boxed{5}$ Asymptotes

ـــ خطوط وهمية تعرفنا ال (root locus) هيمشي في أي اتجاه.

① no. of Asymptotes

$$= \underset{\text{no. of poles}}{n} - \underset{\text{no. of zeros}}{m}$$

② Center of Asymptotes: $C_A = \dfrac{\sum \text{Poles} - \sum \text{zeros}}{n-m}$

③ $\theta = \dfrac{(2L+1)180}{n-m}$

Check stability

$$1 + GH(s) = 0$$

$\boxed{ex}$ $s(s+3)(s+4) + K(s+1) = 0$

$$s^3 + 7s^2 + (12+K)s + K = 0$$

Routh

s^3	1	$12+K$
s^2	7	K
s^1	$\dfrac{7(12+K)-K}{7} > 0$ ①	
s^0	$K > 0$ ②	

① ، ② هما شروط ال (stability)

$$* \ GH(s) = \frac{(K+3)\,3}{(s+3)(s+2)}$$

نرجع للصورة $\boxed{1+GH(s)}$ ونفصل مابين K وال 3.

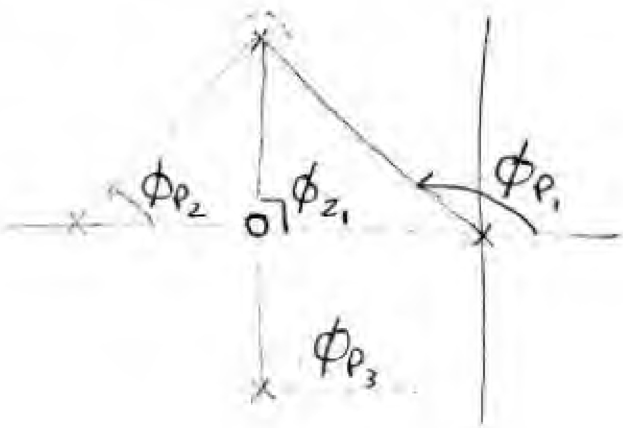
$$* \ GH(s) = \frac{20K(s+3)}{(s+4)(s+5)} \longrightarrow \text{Put } K' = 20K$$

~~Departu~~ (Departure Angle)

$$\theta_D = 180 - \phi_P + \phi_Z$$

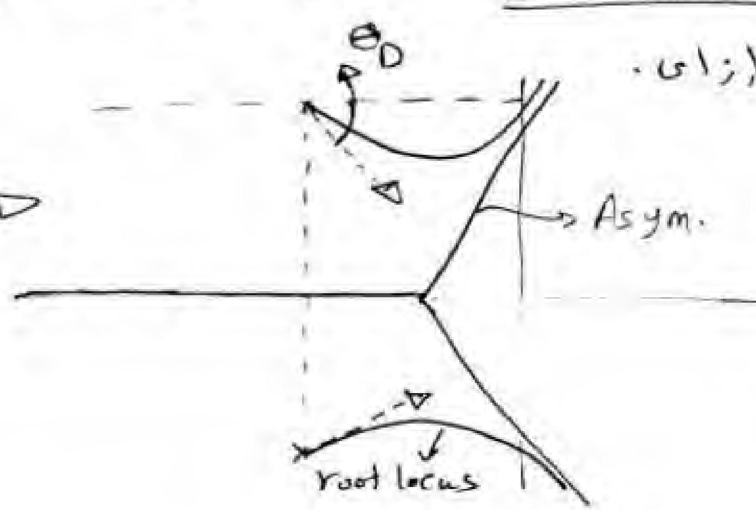
$$\phi_P = \phi_{P_1} + \phi_{P_2} + \phi_{P_3}$$

$$\phi_Z = \phi_{Z_1}$$



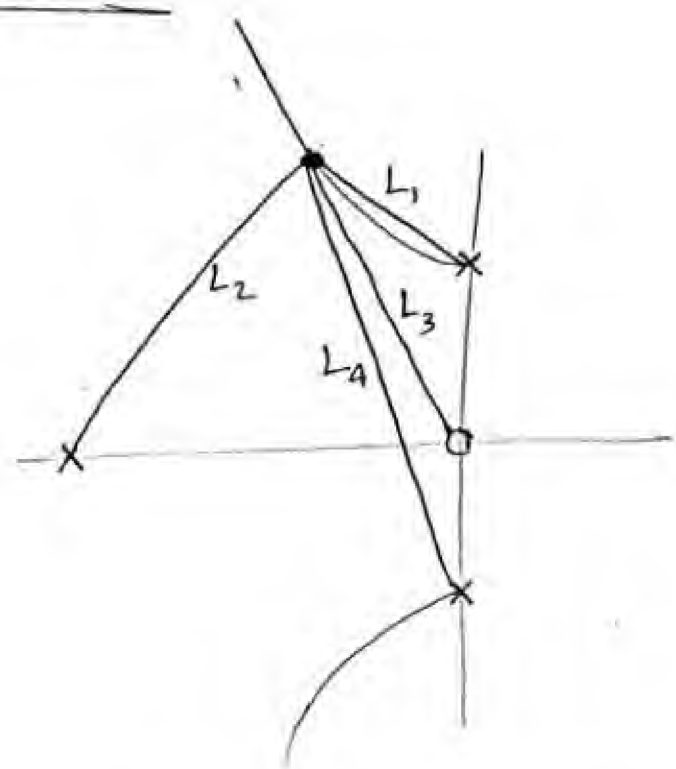
نستخدم ال (Departure Angle) في حالة إننا عندنا (جذر تخيلي) ولا نعلم تحركه ال (root locus) منه هيكون إزاي.

$\boxed{ex} \Rightarrow$



Properties

$\boxed{1}$ Find K at s_o

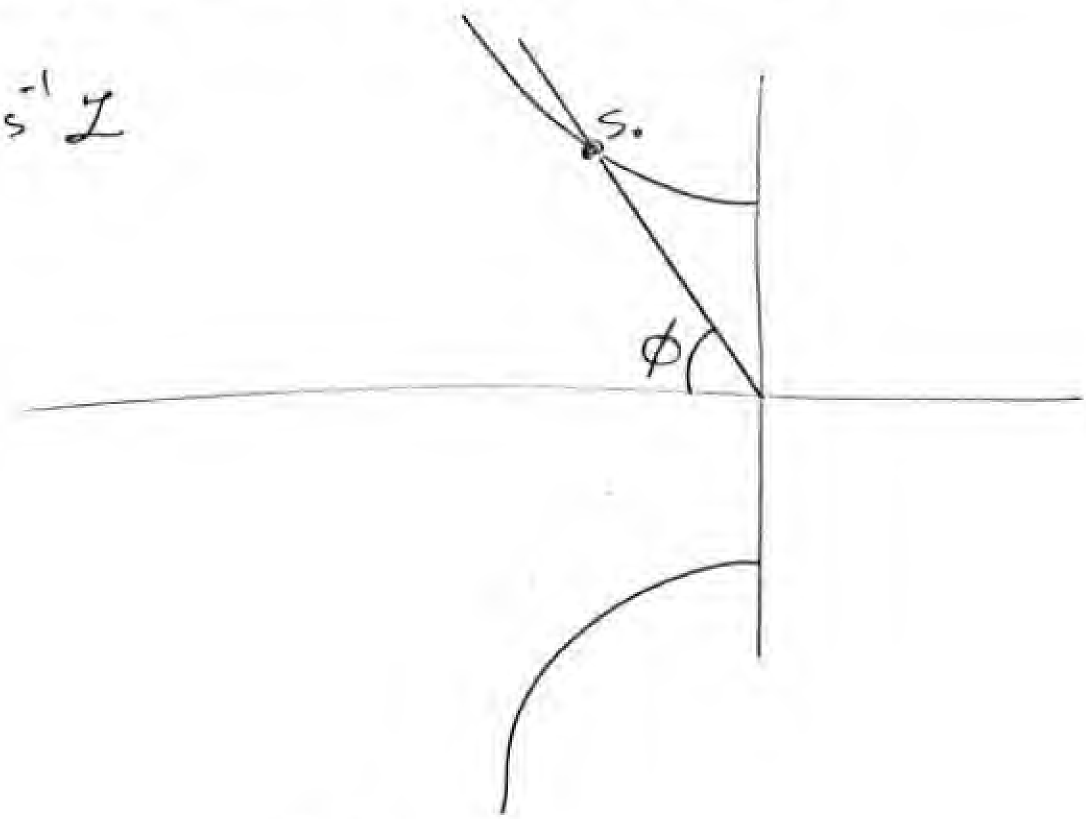


$$K\Big|_{s_o} = \frac{\prod \text{Poles}}{\prod \text{Zero}}$$

$$= \frac{L_1 L_2 L_4}{L_3}$$

$\boxed{2}$ K at $\zeta = 0.5$ (damping ratio)

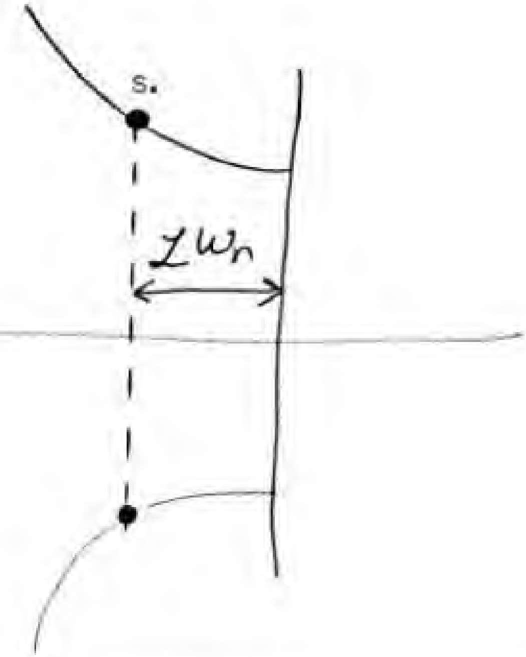
$\phi = \cos^{-1} \zeta$



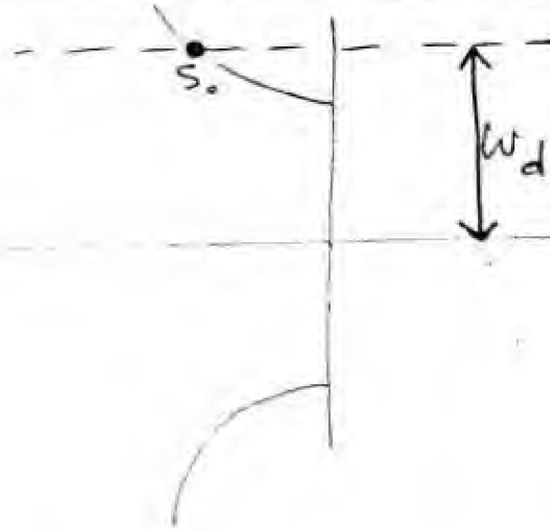
$\boxed{3}$ find K at $t_s = \checkmark$

$$t_s = \frac{4}{\zeta \omega_n}$$

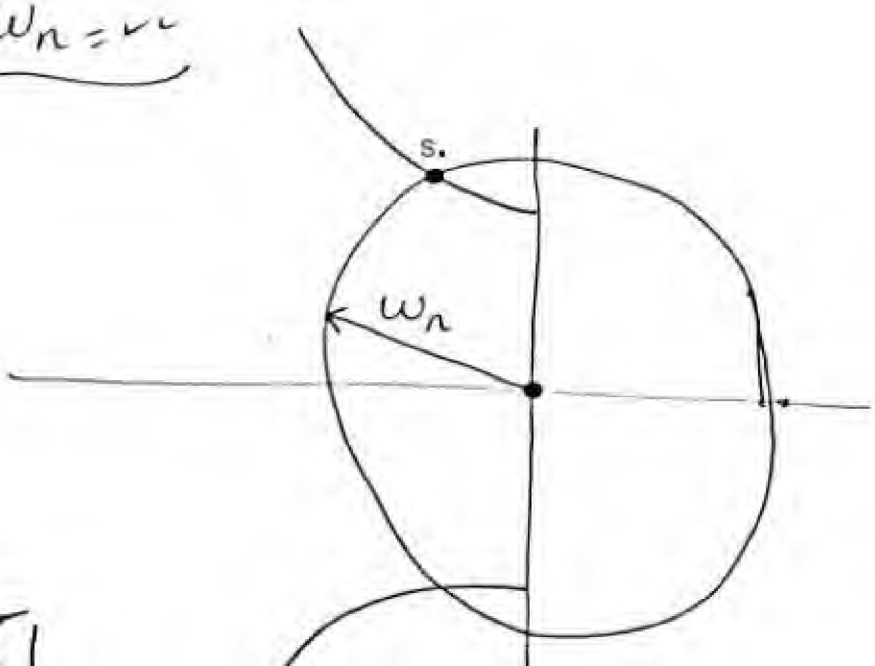
$\Rightarrow \zeta \omega_n = \checkmark$



$\boxed{4}$ K at $\omega_d = \checkmark$



$\boxed{5}$ K at $\cancel{\omega_d}\ \omega_n = \checkmark$



$\boxed{5}$

Bode diagram

given o.L.T.F $GH(s)$ to get bode diagram

① Replace $s \longrightarrow J\omega$

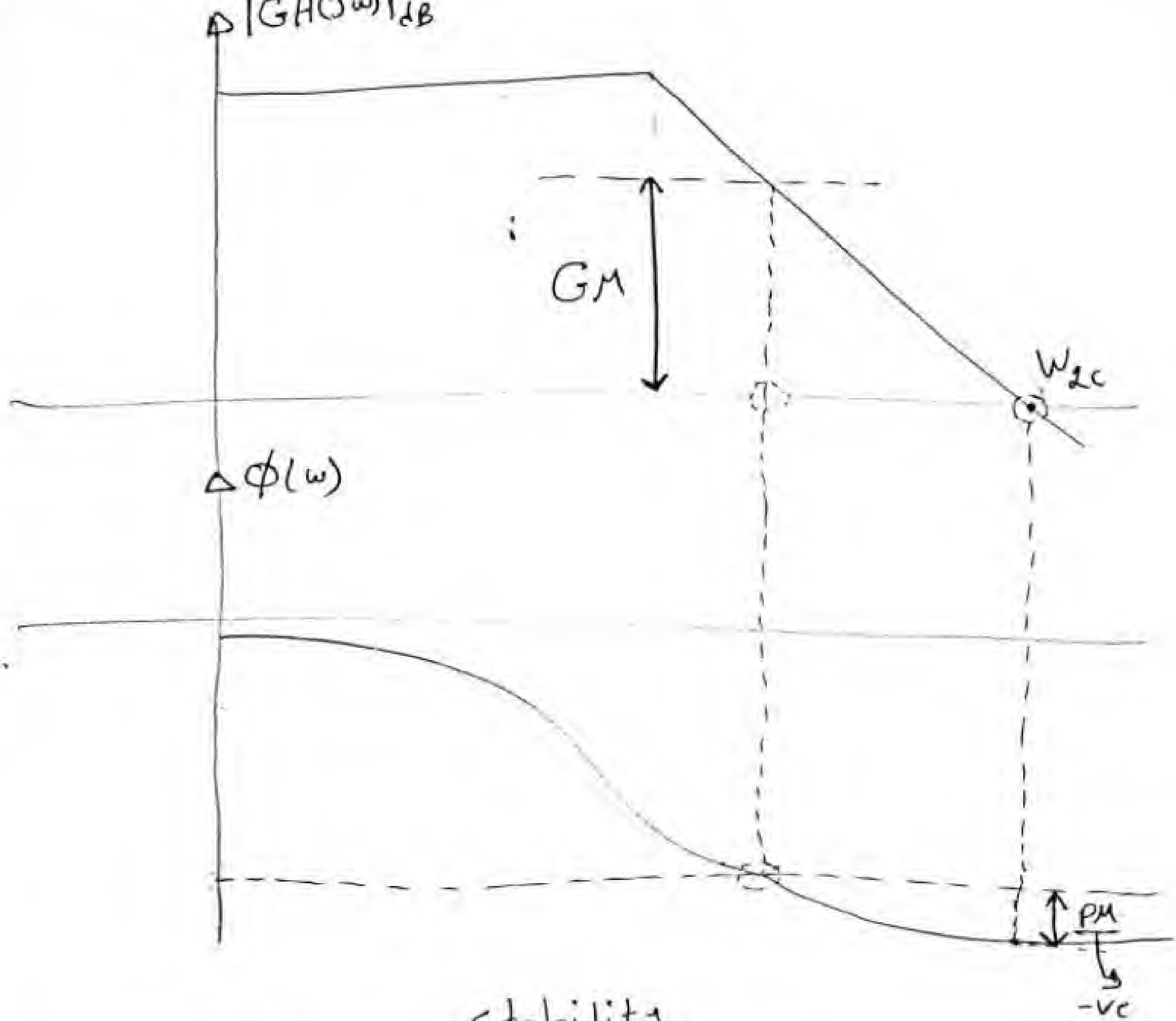
$$GH(J\omega) = GH(s)\Big|_{s \to J\omega}$$

② $\underbrace{|GH(J\omega)|}_{\text{magnitude}}$, $\underbrace{\angle GH(J\omega)}_{\text{Angle}}$

③ $$|GH(J\omega)|_{dB} = 20 \, Log \, |GH(J\omega)|$$

$$|\quad| = \sqrt{(Real)^2 + (imag)^2}$$

$$\phi = \tan^{-1}\left(\frac{imag}{Real}\right)$$



stability

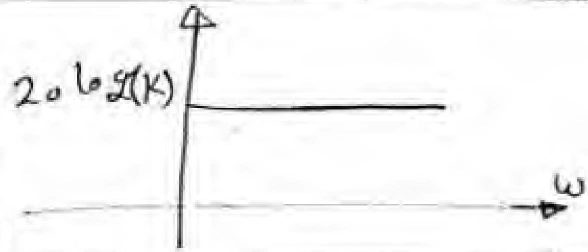
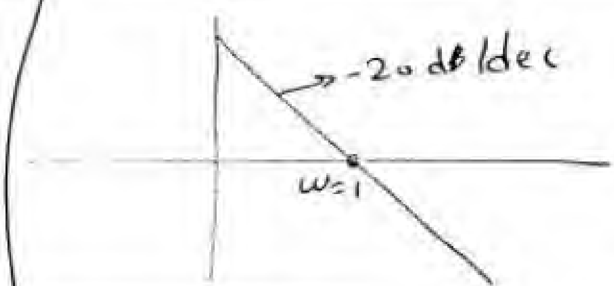
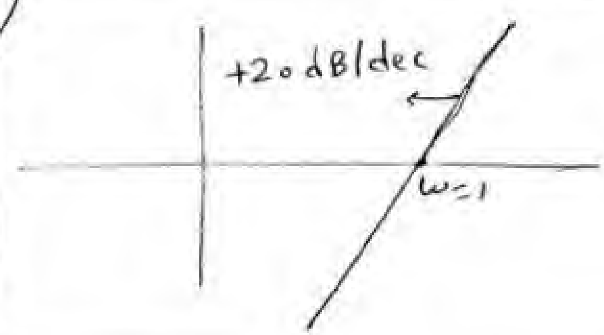
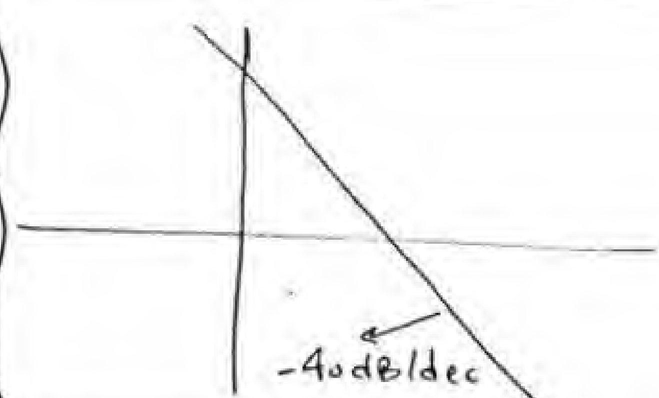
GM, PM

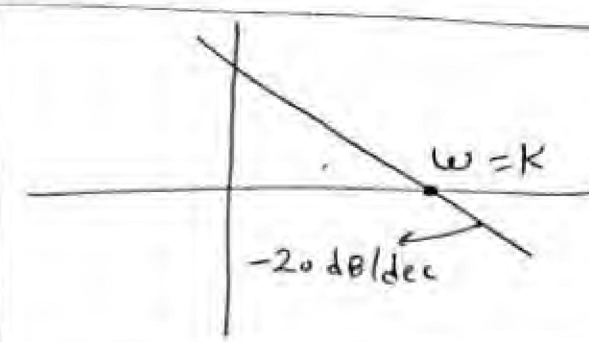
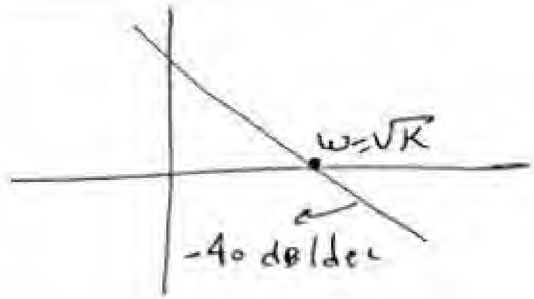
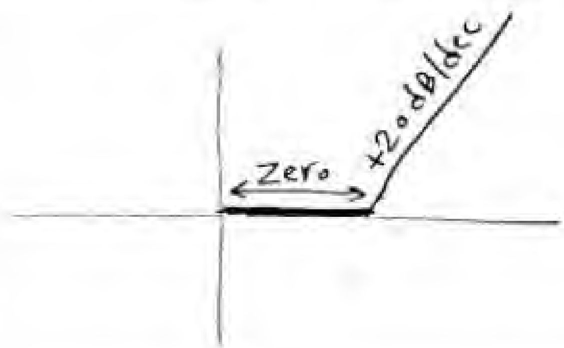
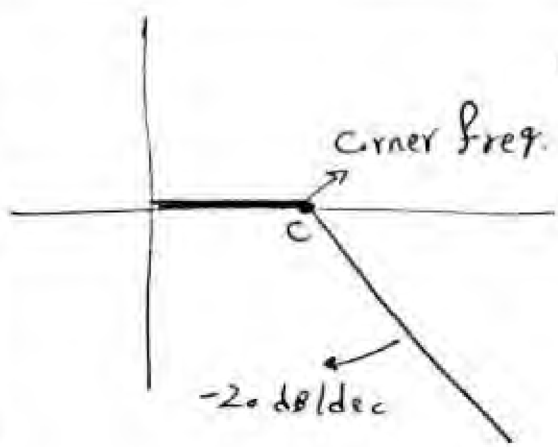
a) > 0 → +ve ⇒ stable

b) < 0 → -ve ⇒ unstable

c) $= 0$ critical stable

$$PM = 180 + \phi(\omega)\Big|_{\omega = \omega_{gc}}$$

Term	$\phi(\omega)$	$\lvert\ \rvert_{dB}$
K	0	
$\frac{1}{s}$ or $\frac{1}{j\omega}$	-90	
s or $j\omega$	$+90$	
$\frac{1}{s^2}$ or $\frac{1}{j\omega \cdot j\omega}$	-180	

Term	$\phi(\omega)$	$\lvert\,\rvert_{dB}$
$\frac{K}{s}$ or $\frac{K}{J\omega}$	-90	
$\frac{K}{s^2}$ or $\frac{K}{J\omega \cdot J\omega}$	-180	
$1 + \frac{s}{c}$	$\tan^{-1}\left(\frac{\omega}{c}\right)$	
$\frac{1}{1 + \frac{s}{c}}$	$-\tan^{-1}\left(\frac{\omega}{c}\right)$	

$$GH(s) = (s)^{\pm n}$$

$\hookrightarrow | \; |_{dB} \longrightarrow$ $\boxed{\omega=1}$ خط مستقيم يمر ب

• البسط ← $\pm 20n \; dB/decade$ ← وميله

المقام ←

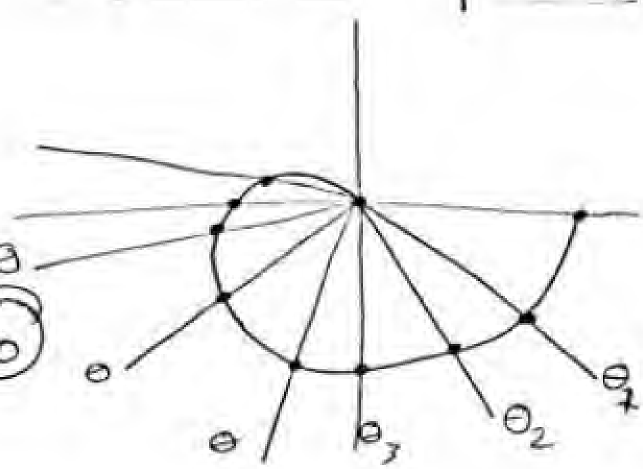
$$\phi(\omega) = \pm 90 n$$

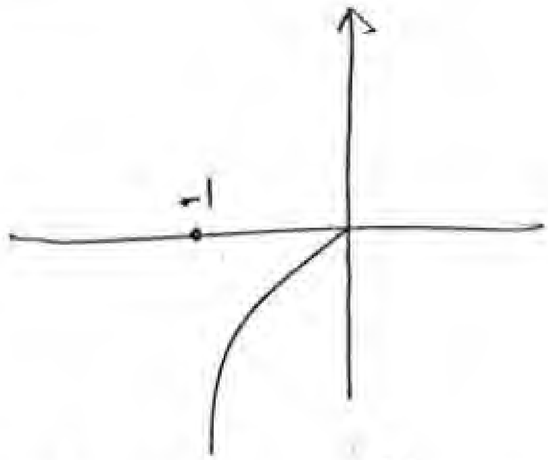
Polar Plot

$\boxed{1}$ $s \longrightarrow j\omega \quad \Rightarrow GH(j\omega)$

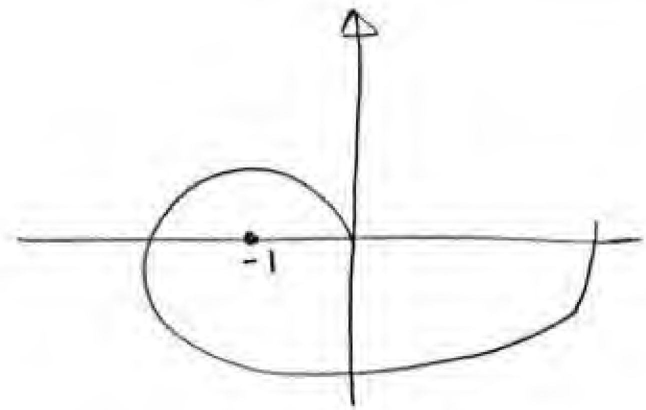
$\boxed{2}$ $|GH(j\omega)| \quad , \; \phi(\omega) = \angle GH(j\omega)$

ω	0	. - - . — - - .	∞
$\lvert GH(j\omega)\rvert$	⋮		
$\phi(\omega)$			





"stable"



"unstable"

ـــ لو المنحنى بداخله (-1) يكون (unstable)

ـــ لو عايز تجيب (GM) تنظر للجدول وتشوف أقرب زاوية لـ $-180°$ وتقلل قيمة ω حتى تصل لـ $-180°$

* using try and error

$\omega = 2 \Rightarrow \phi(\omega) = -184.49$

$\omega = 1.9 \Rightarrow \phi(\omega) = -181.29$

أرقام مثال

$\omega = 1.8 \Rightarrow \phi(\omega) = -177.4°$

$\omega = 1.88 \Rightarrow \phi(\omega) = -180.32°$

$\therefore \omega_{pc} \simeq 1.88$ rad/sec

$$GM = \frac{1}{|GH(j\omega)|_{\omega=\omega_{pc}}}$$

<u>or</u> حل آخر (رياضي)

$$\phi(w) = -\tan^{-1}(2w) - \tan^{-1}(0.5w) - \tan^{-1}(w)$$

$$W_{pc} \Longrightarrow w \quad \text{at} \quad \phi(w) = -180$$

$$-180 = -\tan^{-1}(2w) - \tan^{-1}(0.5w) - \tan^{-1}(w)$$

$$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan x \cdot \tan y}$$

$$180 - \tan^{-1}(2W_{pc}) = \tan^{-1}(0.5W_{pc}) - \tan^{-1}(W_{pc})$$

← نأخذ tan للطرفين

$$\frac{\tan(180) - 2W_{pc}}{1 + \tan(180)(2W_{pc})} = \frac{0.5W_{pc} + W_{pc}}{1 - 0.5\, W_{pc}^2}$$

$$\hookrightarrow W_{pc} = 1.8708 \text{ rad/sec}$$

$$GM = \frac{1}{\left| GH(jw) \right|_{w = W_{pc}}}$$

PM

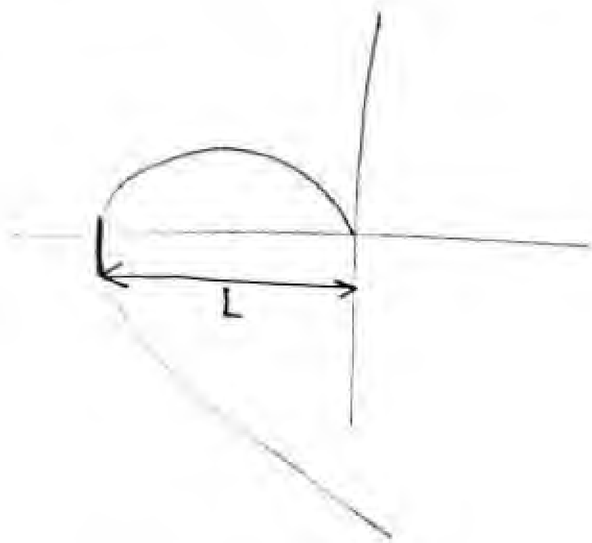
$$PM = 180 + \phi(W_{2c})$$

— في الجدول القيمة التي تساوي $|GH(j\omega)| = 1$

عندها قيمة ω تكون W_{2c} نحسب الزاوية عندها.

$$GM = \frac{1}{L}$$

من أحد طرق حسابه



— في ال G لو لم تجد القيمة (PM) $|GH(j\omega)| = 1$

وفيه مثلاً القيمة 1.43 هنستخدم طريقة

(~~trial~~ try and error) ((Page 11))

* لحد ما توصل $GH(j\omega)$ لقيمة قريبة من الواحد